

# BOUNDARY CONDITIONS FOR THE SLIPPAGE OF A BINARY MIXTURE OF GASES AND THEIR APPLICATION IN THE DYNAMICS OF AEROSOLS.

## 1. FLOW OF A MIXTURE OF GASES ALONG A SOLID PLANE WALL

Yu. I. Yalamov, A. A. Yushkanov, and  
S. A. Savkov

UDC 532.529.5

*We investigate the flow of a thermally and concentrationally inhomogeneous binary mixture of gases near a solid nonevaporating surface. Using the method of half-space moments to solve the kinetic equation with the Boltzmann collision integral, we determine the velocity distribution function of the molecules of the mixture components. The slip velocity of the mixture is calculated.*

Theoretical investigations of the state of inhomogeneous mixtures of gases near the surfaces of bodies immersed in a flow were carried out in a number of works (see, e.g., [1–6]). In most cases use was made of model forms of the collision integral [1–4]. In [5, 6], a kinetic equation with the Boltzmann collision integral was solved. But in [5] the author limited the solution to two moments in the distribution function. The authors of [6] used four moments and considered the specific case of a dilute mixture where the concentration of the molecules of one of the components was small compared to that of the other. Moreover, in [6] an error was made when the moments of the collision integral of discontinuous velocity functions were calculated. It should be noted that in [1, 5, 6] only diffusional slip was considered. There, thermodiffusional effects were not taken into account.

Let us consider the flow of a binary mixture of gases along a solid plane wall in a field of tangential (to its surface) temperature and concentration gradients of the mixture components, provided that the mass velocity of the gas increases uniformly with distance from the wall. It is assumed that changes in the concentration, temperature, and mass velocity of the mixture are small over the mean free path of its molecules  $\lambda$ , and the gradients of the temperature  $T$  and the relative concentration of the first component of the mixture  $n_{10} = n_1/n$  ( $n = n_1 + n_2$ ) coincide in direction with the mass velocity of the mixture  $U$ . The latter assumption does not diminish the generality of the results, since otherwise the slip velocity is merely determined as the vector sum of the velocities of diffusional, thermal, and isothermal slips.

Let us introduce a coordinate system with origin on the wall surface. The  $X$  axis is directed along the normal and the  $Y$  axis along  $U$ . The state of the mixture is described by the velocity distribution function of the molecules of the mixture components

$$f_i(x, \mathbf{v}_i) = f_i^0 \left\{ 1 + \left( \frac{2m_i}{kT} \right)^{1/2} c_{iy} U_y + \psi_{C-E}^i + \psi_B^i + \Phi_i(x, \mathbf{v}_i) \right\}. \quad (1)$$

Here  $\Phi_i(x, \mathbf{v}_i)$  is the correction to for the volumetric distribution describing the influence of the wall. Outside the Knudsen layer, i.e., when  $x \gg \lambda$ ,  $\Phi_i = 0$ :

$$f_i^0 = n_i (m_i/2\pi kT)^{3/2} \exp \left\{ -c_i^2 \right\};$$

$$\psi_{C-E}^i = [D_i^0 + D_i^1 S_{3/2}^1(c_i^2)] c_{iy} \frac{\partial n_{10}}{\partial y} + [A_i^0 +$$

$$+ A_i^1 S_{32}^1(c_i^2) ] c_{iy} \frac{\partial \ln T}{\partial y} + B_i c_{ix} c_{iy} \frac{\partial U_y}{\partial x};$$

$$\psi_B^i = D_B^i c_{ix} c_{iy} \frac{\partial n_{10}}{\partial x \partial y} + [A_{Bi}^0 + A_{Bi}^1 S_{52}^1(c_i^2)] c_{ix} c_{iy} \frac{\partial^2 \ln T}{\partial x \partial y};$$

$$S_v^1(x) = 1 + \nu - x; \quad c_i = (m_i/2kT)^{1/2} v_i.$$

The coefficients  $A_i^0, A_i^1, B_i, D_i^0, D_i^1, A_{Bi}^0, A_{Bi}^1,$  and  $D_B^i$  are given in [7]. The terms in the Chapman-Enskog function  $\psi_{C-E}^i$  involving  $A_i^0$  and  $D_i^1$  describe thermodiffusional effects.

The distribution function (1) takes into account the Barnett terms  $\psi_B^i$ . Generally speaking, Barnett slip is an effect of higher order of smallness, and therefore it can be neglected when determining the gas flow velocity along a plane surface. However, its calculation is of interest in its own right. Moreover, account for Barnett slip is necessary when considering a flow of gas past a curved surface, which will be the subject of the next paper.

The correction  $\Phi_i(x, v_i)$  is determined from the solution of the Boltzmann linearized equation [7]

$$f_i^0 v_{ix} \frac{\partial \Phi_i}{\partial x} = - n_i^2 I_{ii}(\Phi_i) - n_1 n_2 I_{12}(\Phi_1 - \Phi_2) \quad (2)$$

( $I_{ij}$  is the linearized collision integral defined in [7]).

In accordance with the traditional procedure of the method of half space-moments [1-6], we expand  $\Phi_i$  in a series in half-space velocity polynomials. Limiting ourselves to four moments, we write

$$\Phi_i(x, v_i) = \sum_{l=0}^3 a_l^i(x) P_l(v_i), \quad (3)$$

$$P_0 = c_y, \quad P_1 = c_y \operatorname{sign} c_x, \quad P_2 = c_x c_y, \quad P_3 = c_x c_y \operatorname{sign} c_x,$$

$$\operatorname{sign} x = 1 (x > 0), \quad \operatorname{sign} x = -1 (x < 0).$$

The coefficients  $a_l^i(x)$  are determined by solving a system of differential moment equations constructed by multiplying Eq. (2) by  $P_i$  and integrating over the entire space of velocities:

$$\left( \frac{2kT}{m_i} \right)^{1/2} \sum_l \frac{\partial a_l^i}{\partial x} J_{kl}^i = - \sum_l \left( n_i \omega_{kl}^i + \sum_{j=1}^2 n_j \gamma_{kl}^{ij} \right), \quad (4)$$

where

$$J_{kl}^i = \frac{1}{n_i} \int f_i^0 P_k(c_i) P_l(c_i) dv_i,$$

$$\omega_{kl}^i = [P_k(c_i), P_l(c_i)]_i, \quad \gamma_{kl}^{ij} = [P_k(c_i), P_l(c_j)]_{12}.$$

The notation of the bracketed integrals corresponds to the traditional one adopted in [7]. Analytical expressions for these integrals in the case of molecules interacting as rigid spheres are given in the Appendix.

The solution of system of equations (4) has the form

$$a_l^i = \sum_{\nu=1}^3 C_\nu \alpha_{l\nu}^i \exp \{ -\gamma_\nu x \}, \quad (5)$$

Here  $\gamma_\nu$  is determined from the condition of the determinant of the system of algebraic equations obtained upon substitution of solutions (5) into Eqs. (4) being equal to zero. In this case, the positive roots are retained. The coefficients  $\alpha_{b\nu}^i$  are expressed via determinants of rank 7. The integration constants  $C_\nu$  are determined from kinetic boundary conditions for the reflection of gas molecules from the wall:

$$v_{ix} f_i^+ = \Omega f_i^-, \quad (6)$$

$$\Omega f_i^- = - \int_{v'_{ix} < 0} v'_{ix} R(v'_i, v_i) f_i^-(v'_i) dv'_i,$$

where  $R(v'_i, v_i)$  is the probability that a molecule with the velocity  $v'_i$  acquires the velocity  $v_i$  as a result of reflection from the wall. The superscripts "-" and "+" denote distribution functions of molecules incident on and reflected from the wall, respectively.

Following the procedure developed in [8, 9], we present conditions (6) in moment form. For this, we multiply them by  $v_{iy}$  and  $v_{ix}v_{iy}$  and integrate over the positive half-space of velocities:

$$\int_{v_{ix} > 0} v_{ix} v_{iy} f_i^+ dv_i = \int_{v_{ix} > 0} v_{iy} \Omega f_i^- dv_i, \quad (7)$$

$$\int_{v_{ix} > 0} v_{ix}^2 v_{iy} f_i^+ dv_i = \int_{v_{ix} > 0} v_{ix} v_{iy} \Omega f_i^- dv_i.$$

Substituting the distribution function (1), (3), (5) into Eqs. (7) and integrating, we obtain a system of algebraic equations for determining the integration constants and the slip velocity of the mixture, the solution of which yields

$$U_y = K_{DSl} D_{12} \frac{\partial n_{10}}{\partial y} + K_{TSl} \frac{\eta}{\rho T} \frac{\partial T}{\partial y} + K_{Sl} \lambda \frac{\partial U_y}{\partial x} + K_{DSl}^B D_{12} \lambda \frac{\partial^2 n_{10}}{\partial x \partial y} + K_{TSl}^B \frac{\eta \lambda}{\rho T} \frac{\partial^2 T}{\partial x \partial y}.$$

Here, the coefficients of mutual diffusion  $D_{12}$  and dynamic viscosity  $\eta$  of the mixture, which are calculated in the first approximation of the Chapman-Enskog method [7], are separated;  $\lambda = (\pi/2\rho nkT)^{1/2}$ ;  $\rho = n_1 m_1 + n_2 m_2$ .

The values of the kinetic coefficients calculated for specific relationships between the masses, diameters, and concentrations of the molecules of the mixture components when the molecules are represented by rigid spheres in the case of purely diffuse reflection from the wall ( $\Omega f_i^- = v_{ix} f_i^+$ ) are given in Tables 1-7, where  $M_i = m_i/(m_1 + m_2)$ ;  $R_i = \sigma_i^2/(\sigma_1 + \sigma_2)$ ;  $\sigma_i$  is the effective diameter of the molecules of the  $i$ -th mixture component. All the calculations in the tables are given for the solid-sphere model of the molecules on condition of purely diffuse reflection from the surface.

Attention should be paid to the dependence of the diffusional slip coefficient of a dilute mixture on the relationship between the diameters of the molecules of its components (Table 1). In the region  $R_1 < 0.6$  for  $M_1 < 0.6$  we observe a decrease in  $K_{DSl}$  with  $R_1$ . Such a behavior of the dependence of  $K_{DSl}$  on  $R_1$  is due to the contribution of the thermodiffusional terms in the Chapman-Enskog function to the slip velocity. Neglect of the indicated terms leads to a qualitative change in this dependence, i.e., to growth of  $K_{DSl}$  with a decrease in  $R_1$ . In particular, for  $R_1 = 0.2$  and  $M_1 = 0.1$  the quantity  $K_{DSl}$  is equal to 7.52. We can also note that with other relationships between the masses, diameters, and concentrations of the molecules of the mixture components the contribution of thermodiffusional effects to the diffusional slip velocity amounts to at least 10%.

Of particular interest is the calculation of the slip velocity due to the Barnett terms in the distribution function (Tables 3 and 4). In [10, 11] the Barnett slip velocity is calculated as

$$U_y^B = - \frac{\lambda}{\eta} K_{Sl} \sigma_{xy}^B, \quad (8)$$

TABLE 1. Values of  $K_{DSI}$  at  $n_{20} = 0.001$

$M_1$	$R_1$								
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.1	2.08	5.76	6.03	5.63	4.93	4.01	2.89	1.58	0.11
0.2	-1.42	1.70	2.06	1.95	1.64	1.20	0.66	0.02	-0.71
0.3	-1.79	0.59	0.89	0.84	0.64	0.36	0.01	-0.40	-0.87
0.4	-1.40	0.21	0.40	0.36	0.21	0.01	-0.25	-0.54	-0.88
0.5	-0.79	0.10	0.18	0.12	0	-0.16	-0.36	-0.59	-0.84
0.6	-0.21	0.08	0.07	-0.00	-0.11	-0.24	-0.40	-0.57	-0.78
0.7	0.17	0.08	0.01	-0.07	-0.16	-0.27	-0.39	-0.53	-0.69
0.8	0.27	0.04	-0.04	-0.11	-0.19	-0.27	-0.36	-0.47	-0.59
0.9	0.12	-0.02	-0.08	-0.13	-0.17	-0.23	-0.29	-0.36	-0.44

TABLE 2. Values of  $K_{DSI}$  at  $n_{10} = n_{20} = 0.5$

$M_1$	$R_1$								
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.1	1.00	0.87	0.74	0.60	0.46	0.35	0.26	0.19	1.13
0.2	0.96	0.84	0.70	0.56	0.41	0.28	0.17	0.08	0.01
0.3	0.81	0.70	0.58	0.44	0.29	0.15	0.03	-0.07	-0.15
0.4	0.64	0.55	0.43	0.29	0.15	0.01	-0.12	-0.23	-0.31
0.5	0.48	0.39	0.27	0.14	0	-0.14	-0.27	-0.39	-0.48

TABLE 3. Values of  $K_{DSI}^B$  at  $n_{20} = 0.001$

$M_1$	$R_1$								
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.1	-19.51	-18.78	-16.99	-13.44	-7.11	3.27	19.25	42.60	75.36
0.2	-8.27	-7.93	-7.14	-5.64	-3.06	1.08	7.34	16.40	29.03
0.3	-4.56	-4.33	-3.84	-2.92	-1.37	1.07	4.72	9.97	17.24
0.4	-2.74	-2.58	-2.22	-1.57	-0.49	1.18	3.67	7.22	12.13
0.5	-1.69	-1.59	-1.29	-0.80	0	1.24	3.07	5.68	9.26
0.6	-1.02	-0.92	-0.71	-0.33	0.29	1.24	2.64	4.62	7.34
0.7	-0.58	-0.50	-0.34	-0.04	0.43	1.16	2.24	3.76	5.85
0.8	-0.29	-0.23	-0.11	0.11	0.46	1.01	1.81	2.95	4.50
0.9	-0.10	-0.06	0.01	0.16	0.39	0.74	1.27	2.02	3.04

TABLE 4. Values of  $K_{DSI}^B$  at  $n_{10} = n_{20} = 0.5$

$M_1$	$R_1$								
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.1	-6.04	-4.58	-3.14	-1.90	-0.93	-0.24	0.21	0.51	0.69
0.2	-5.60	-4.44	-3.19	-1.98	-0.93	-0.08	0.56	1.01	1.32
0.3	-4.86	-3.92	-2.85	-1.74	-0.70	0.22	0.96	1.54	1.97
0.4	-4.09	-3.31	-2.39	-1.39	-0.37	0.58	1.42	2.11	2.64
0.5	-3.34	-2.70	-1.90	-0.99	0	0.99	1.90	2.70	3.34

where  $\sigma_{xy}^B$  is the stress tensor due to the Barnett terms in the distribution function. For a simple gas this approach gives  $K_{TSI}^B = -3$ ,  $K_{SI} = -3.44$ . The difference from the results of a direct calculation ( $K_{TSI}^B = -3.57$ ) amounts to 3%. In the case of a mixture of gases this difference for  $K_{TSI}^B$  usually does not exceed 10%. For the diffusional slip velocity the difference is much greater. For example, according to Eq. (8), when  $n_{20} = 0.001$ ,  $M_1 = 0.1$ ,  $R_1 = 1.8$ , we obtain  $K_{DSI}^B = 11.4$ , which differs from the corresponding value in Table 3 by almost an order of magnitude.

TABLE 5. Values of  $K_{TSI}$  at  $n_{10} = n_{20} = 0.5$

$M_1$	$R_1$								
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.1	1.35	1.22	1.10	0.99	0.90	0.83	0.76	0.71	0.66
0.2	1.30	1.21	1.13	1.04	0.97	0.91	0.86	0.81	0.78
0.3	1.28	1.23	1.17	1.11	1.06	1.01	0.97	0.94	0.91
0.4	1.24	1.22	1.19	1.16	1.13	1.10	1.08	1.06	1.05
0.5	1.16	1.16	1.16	1.15	1.15	1.15	1.16	1.16	1.16

TABLE 6. Values of  $K_{TSI}^B$  at  $n_{10} = n_{20} = 0.5$

$M_1$	$R_1$								
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.1	-8.89	-6.84	-5.17	-4.06	-3.45	-3.17	-3.02	-2.91	-2.84
0.2	-6.74	-5.60	-4.58	-3.82	-3.38	-3.19	-3.13	-3.13	-3.15
0.3	-5.76	-5.02	-4.32	-3.77	-3.45	-3.34	-3.38	-3.47	-3.57
0.4	-5.12	-4.62	-4.13	-3.74	-3.53	-3.52	-3.66	-3.86	-4.06
0.5	-4.58	-4.25	-3.92	-3.66	-3.57	-3.66	-3.92	-4.25	-4.58

TABLE 7. Values of  $K_{SI}$  at  $n_{10} = n_{20} = 0.5$

$M_1$	$R_1$								
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.1	1.49	1.39	1.32	1.29	1.30	1.34	1.38	1.43	1.47
0.2	1.41	1.33	1.26	1.22	1.22	1.24	1.29	1.34	1.39
0.3	1.36	1.29	1.22	1.18	1.17	1.20	1.24	1.29	1.34
0.4	1.33	1.27	1.21	1.17	1.15	1.17	1.21	1.27	1.32
0.5	1.32	1.26	1.20	1.16	1.15	1.16	1.20	1.26	1.32

## APPENDIX

In addition to what has been said in the beginning of this article, below we give expressions for the two moments of the collision integral of the discontinuous velocity functions for the solid-sphere model of the molecules, in the calculation of which an error was made in [6]:

$$\begin{aligned}
 & [c_{1y} \operatorname{sign} c_{1x}, c_{2y} \operatorname{sign} c_{2x}]_{12} = \\
 & = I_0 \sqrt{M_1 M_2} \left\{ -\frac{8}{3} - \frac{1}{2\sqrt{M_1 M_2}} + \frac{1}{\sqrt{M_{\max}}} (M_{\max} + \right. \\
 & \quad \left. + 1 - \sqrt{M_1 M_2}) + \frac{1}{(M_1 M_2)^{3/2}} \left[ \frac{2}{3} (M_1^{3/2} + \right. \right. \\
 & \quad \left. \left. + M_2^{3/2}) - M_1^{5/2} - M_2^{5/2} + \frac{1}{3} (M_1^3 + M_2^3) \right] \right\}, \tag{A.1}
 \end{aligned}$$

$$\begin{aligned}
 & [c_{1y} c_{1x} \operatorname{sign} c_{1x}, c_{2y} c_{2x} \operatorname{sign} c_{2x}]_{12} = \\
 & = I_0 \left\{ -\frac{16}{15} M_1 M_2 - \frac{1}{2} \sqrt{M_1 M_2} + \right. \\
 & \quad \left. + \frac{5}{6} \frac{M_1^3 + M_2^3}{\sqrt{M_1 M_2}} - \frac{2}{5} \frac{M_1^5 + M_2^5}{(M_1 M_2)^{3/2}} - \right.
 \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} \sqrt{\left(\frac{M_1 M_2}{M_{\max}}\right)} [1 + 4M_{\max} - 2M_{\max}^2 - \\
& - 2(1 + M_{\max}) \sqrt{M_1 M_2}] + \frac{M_1 M_2}{2} \left[ \frac{1}{M^{5/2}} \left( \frac{2}{15} - \right. \right. \\
& \left. \left. - \frac{M_1}{3} - M_1^2 + 2M_1^3 \right) + \frac{1}{M_1^{5/2}} \left( \frac{2}{15} - \frac{M_2}{3} - M_2^2 + 2M_2^3 \right) \right] + \\
& + \frac{\sqrt{M_1 M_2}}{4} (M_1 - M_2) \left[ \frac{M_2}{M_1^2} (1 + 2M_2^{3/2} + 3M_2) - \frac{M_1}{M_2^2} (1 + 2M_1^{3/2} - 3M_1) \right] \}. \quad (A.2)
\end{aligned}$$

For the indicated moments of the collision integrals the authors of [6] obtained analytical expressions that diverge in the limiting cases  $M_1 \rightarrow 0$  and  $M_2 \rightarrow 0$ . There is no such divergence in (A1) and (A2) and it should not be present from both physical and mathematical viewpoints. Note also that the values of the moments of the collision integrals (A1) and (A2) coincide with the results given by Breton in [5], and therefore the criticism of Breton in [6] is not justifiable and is misleading.

## NOTATION

$\lambda$ , mean free path of mixture molecules;  $U$ , mass velocity of the mixture;  $T$ , temperature;  $D_{12}$  and  $\eta$ , coefficients of mutual diffusion and viscosity of the mixture;  $k$ , Boltzmann constant;  $n_i$ ,  $m_i$ ,  $\sigma_i$  and  $v_i$ , concentration, mass, effective diameter, and intrinsic velocity of molecules of the  $i$ -th component of the mixture, respectively;  $n_{i0}$ ,  $R_i$ , and  $M_i$ , relative concentration, diameter, and mass of molecules;  $c_i$ , dimensionless intrinsic velocity of molecules of the  $i$ -th component of the mixture;  $f_i$ , velocity distribution function of mixture molecules;  $f_i^0$ , equilibrium Maxwellian distribution function;  $\psi_{C-E}^i$  and  $\psi_B^i$ , Chapman-Enskog and Barnett functions;  $\Phi_i$ , correction to the distribution function to describe the effect of the wall;  $K_{DSl}$ ,  $K_{TSl}$ , and  $K_{Sl}$ , coefficients of diffusional, thermal, and isothermal slip;  $K_{DSl}^B$  and  $K_{TSl}^B$ , coefficients of Barnett slip.

## REFERENCES

1. I. N. Ivchenko and Yu. I. Yalamov, *Mekh. Zhid. Gaza*, No. 4, 22-26 (1971).
2. Yu. I. Yalamov, A. A. Yushkanov, and M. N. Gaidukov, *Zh. Tekh. Fiz.*, **45**, 203-205 (1975).
3. Yu. I. Yalamov, M. N. Gaidukov, and A. A. Yushkanov, *Inzh.-Fiz. Zh.*, **29**, No. 3, 489-493 (1975).
4. V. I. Zhdanov and R. V. Smirnova, *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 5, 103-115 (1978).
5. J. P. Breton, *Phys. Fluids*, **12**, No. 10, 2019-2026 (1969).
6. S. P. Bakanov and V. I. Roldugin, *Inzh.-Fiz. Zh.*, **40**, No. 5, 807-817 (1981).
7. S. Chapman and T. Cowling, *Mathematical Theory of Inhomogeneous Gases* [Russian translation], Moscow (1960).
8. A. A. Yushkanov, S. A. Savkov, and Yu. I. Yalamov, *Inzh.-Fiz. Zh.*, **51**, No. 4, 687-688 (1986).
9. S. A. Savkov and A. A. Yushkanov, *Mekh. Zhid. Gaza*, No. 5, 149-152 (1986).
10. S. P. Bakanov and V. I. Roldugin, *Kolloidn. Zh.*, **42**, No. 6, 1043-1050 (1980).
11. S. K. Layalka, *Phys. Fluids*, **14**, No. 12, 2599-2604 (1971).